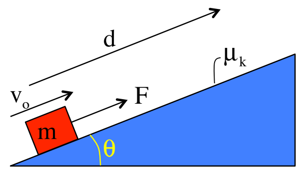
Problem 8.16

A crate is pushed up an incline.

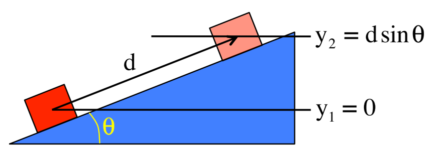


a.) How much work does gravity do on the crate as it moves 5.00 meters up the incline?

There are three ways to do this, the easy way, the easy way made hard and the hard way. The hard way is using $W_{mg} = \vec{F}_{mg} \bullet \vec{d}$. This is nasty, as is the case most of the time, because determining the angle can be a little tricky (for this situation, it's $\theta + 90^\circ$). The easy way is to use the *potential energy function* for gravity near the earth's surface. And the easy way made hard is to use the potential energy function but instead of writing it out from scratch, you try to psych it out and jump at it and mess it up in the process (I can't tell you how many students have done this over the years). We'll do it the easy way using the *potential energy function* first, then for the sake of humor and review do it via the work definition.

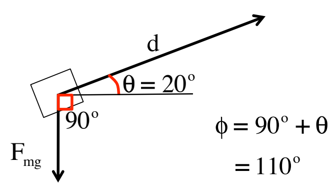
So onward!

If we take the "y = 0" level to be where the body is first viewed, the y-coordinate for the second position will be "$d\sin\theta$" (see sketch). Using that, we can write:

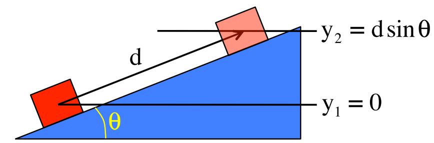


$$\begin{aligned} W_{grav} &= -\Delta U_{grav} \\ &= -\left(mgy_2 - mgy_1\right) \quad (mgy_1 = 0) \\ &= -mg\left(d\sin\theta\right) \\ &= -(10.0\text{ kg})\left(9.80\text{ m/s}^2\right)\left[(5.00\text{ m})\sin 20^\circ\right] \\ &= -168\text{ J} \end{aligned}$$

Using the definition of work, the only hard part is in determine the angle ϕ between the the displacement vector and gravity. Because it is educational to remind yourself how to do that, it is shown to the right.



With the angle, the work gravity does over the motion can also be determined using:



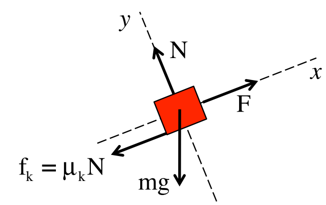
$$\begin{aligned} W_{grav} &= \vec{F}_{grav} \bullet \vec{d} \\ &= \left|\vec{F}_{grav}\right|\left|\vec{d}\right|\cos\phi \\ &= (mg)(d)\cos\phi \\ &= (10.0\text{ kg})\left(9.80\text{ m/s}^2\right)(5.00\text{ m})\left(\cos 110^\circ\right) \\ &= -168\text{ J} \end{aligned}$$

Pretty cool, eh?

b.) What is the increase in *internal energy* due to friction (translation: how much work does friction pull out of the system and dump into the surrounding area in the form of heat)?

To begin with, we need to derive an expression for the normal force. Although by now this should be obvious, for the sake of review we will use N.S.L. to do that on the next page.

f.b.d. on the block:



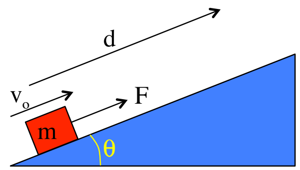
$\Rightarrow \quad N = mg\cos\theta$

So:

$$\begin{aligned} W_f &= \vec{F}_f \bullet \vec{d} \\ &= \left(\left|\vec{F}_f\right|\right)\left|\vec{d}\right|\cos\phi \\ &= \left(\mu_k N\right)(d)\cos 180^\circ \\ &= \left(\mu_k\left(mg\cos\theta\right)\right)(d)\cos 180^\circ \\ &= (.400)(10.0\text{ kg})\left(9.80\text{ m/s}^2\right)\left(\cos 20^\circ\right)(5.00\text{ m})(-1) \\ &= -184\text{ J} \end{aligned}$$

This means that 184 joules of energy are pulled out of the crate system and deposited as heat into the surrounding area.

c.) How much work does “F” do?



$$W_F = \vec{F} \bullet \vec{d}$$
$$= \left(\left|\vec{F}\right|\right)\left|\vec{d}\right|\cos 0^\circ$$
$$= (100.\ \text{N})(5.00\ \text{m})$$
$$= 500.\ \text{J}$$

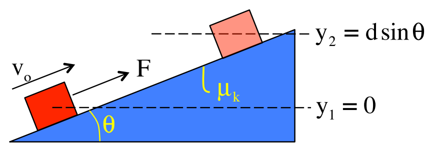
d.) What is the change of kinetic energy of the crate?

This is sort of a *use your head* and *think* kind of problem. Noting the relationships listed, that shouldn't be too hard.

$$W_{net} = W_f + W_{grav} + W_F \qquad \text{and}$$
$$W_{net} = \Delta KE$$
$$\Rightarrow \quad \Delta KE = W_f + W_{grav} + W_F$$
$$\Rightarrow \quad \Delta KE = (-184\ \text{J}) + (-168\ \text{J}) + (500\ \text{J})$$
$$= 148\ \text{J}$$

e.) What is the “final” speed of the crate?

My inclination is to do this using *Conservation of Energy* because that is what this chapter is about . . . and because it's fun. That calculation is shown below.



$$\sum KE_1 + \sum U_1 + \sum W_{ext} = \sum KE_2 + \sum U_2$$
$$\frac{1}{2}mv_o^2 + 0 + (W_F + W_f) = \frac{1}{2}mv_2^2 + mg(y_2)$$
$$\Rightarrow \quad v_2 = \left[\frac{mv_o^2 + 2(W_F + W_f) - 2mg(d\sin\theta)}{m}\right]^{1/2}$$
$$\Rightarrow \quad v_2 = \left[\frac{(10.0\ \text{kg})(1.50\ \text{m/s})^2 + 2\left[(500.\ \text{J}) + (-184\ \text{J})\right] - 2(10.0\ \text{kg})(9.80\ \text{m/s}^2)(5\sin 20^\circ)}{(10.0\ \text{kg})}\right]^{1/2}$$
$$= 5.65\ \text{m/s}$$

We could, though, have used the information from the previous section.

e.) (Cont'd)

That is, knowing the *change of kinetic energy*, the mass and the initial velocity, we could as easily have written:

$$\Delta KE = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_o^2$$
$$\Rightarrow \quad (148\ \text{J}) = \frac{1}{2}(10.0\ \text{kg})v_2^2 - \frac{1}{2}(10.0\ \text{kg})(1.50\ \text{m/s})^2$$
$$\Rightarrow \quad v_2 = \left[\frac{2(148\ \text{J}) + (10.0\ \text{kg})(1.50\ \text{m/s})^2}{(10.0\ \text{kg})}\right]^{1/2}$$
$$= 5.64\ \text{m/s}$$

Same answer. Both ways are acceptable!